Mechanical Calculator
"Consul", the Educated Monkey

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Multiplication is not only a challenge for today's seven-year-old child – 100 years ago most children just had to "push through" and learn the tables by rote. But even in those days, resourceful educators and tinkerers had thought about how to make the learning process a little more enjoyable, intuitive and attractive to children. In 1915 William H. Robertson found a fascinating solution: "Consul".

Story

"Consul, The Educated Monkey" is a mechanical calculation aid for the addition and multiplication of two integers from 1 to 12 originally from the year 1915 (fig. 1). It is easy to use: if the feet of the monkey are adjusted so that they point to the summaries or factors, the result is displayed in the window between the hands of the monkey. If you would like to try out the "live" function, the Flash-Simulation from tan-gram.de[1]. Windows users might also like to see the implementation by Reinhard Atzbach [2].
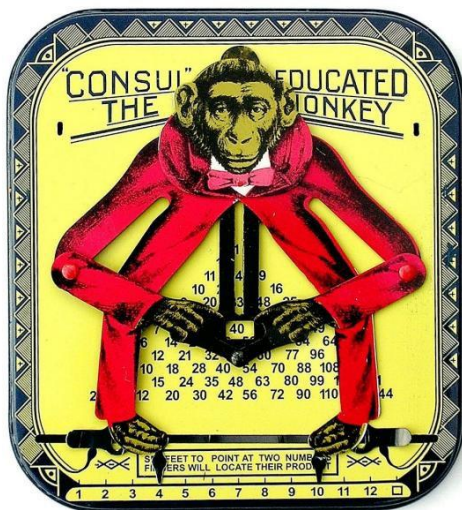

Fig. 1: Consul, the Educated Monkey (Educational Novelty Company, 1915)

"Consul, The Educated Monkey" was patented in 1915 by the American William H. Robertson from Belmont, Ohio. On 27.06.1916 the patent was granted to the "Educated Monkey" (fig. 2); Two and a half years later, on 26.11.1918, he received a second patent on mechanics (fig. 3).
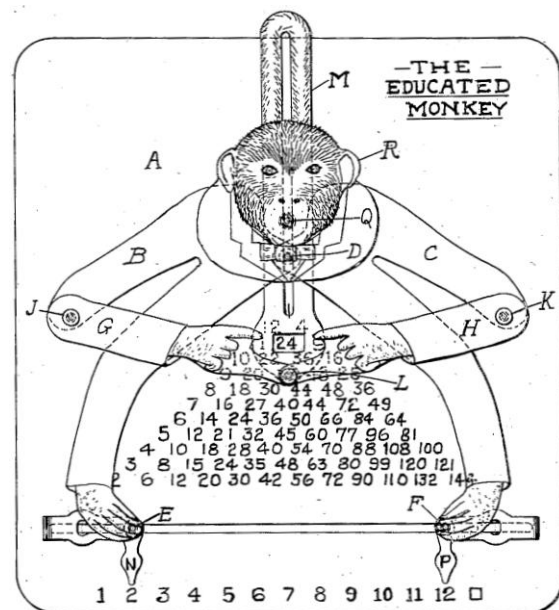

Fig. 2: Educated Monkey (US-Patent No. 1188490 from 27/06/1916 [4])

The 'playful toy' made of sheet metal, sold by his Educational Novelty Company from 1915 to 1922, is today a sought-after collector's item [5].
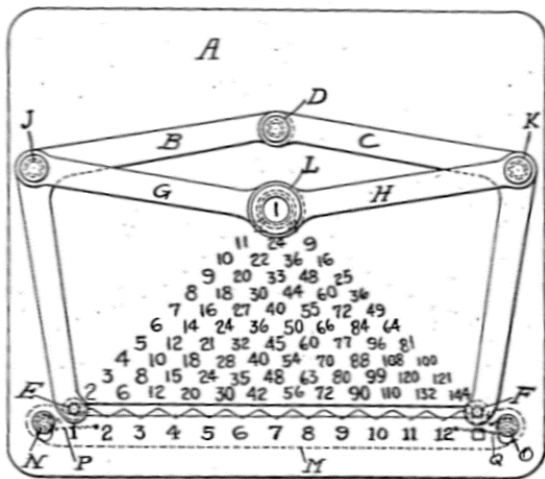
Fig. 3: Mechanics of the Educated Monkey (US-Patent No. 1286112 from 26/11/1918) [4]

There is only speculation about the background of the name 'Consul'. A very plausible one traces him back to the documentary "Consul (the Great) Crosses the Atlantic" the film pionier *Charles Urban* (1867-1942) from 1909, which shows the first visit of a well-known monkey named Consul in the USA.
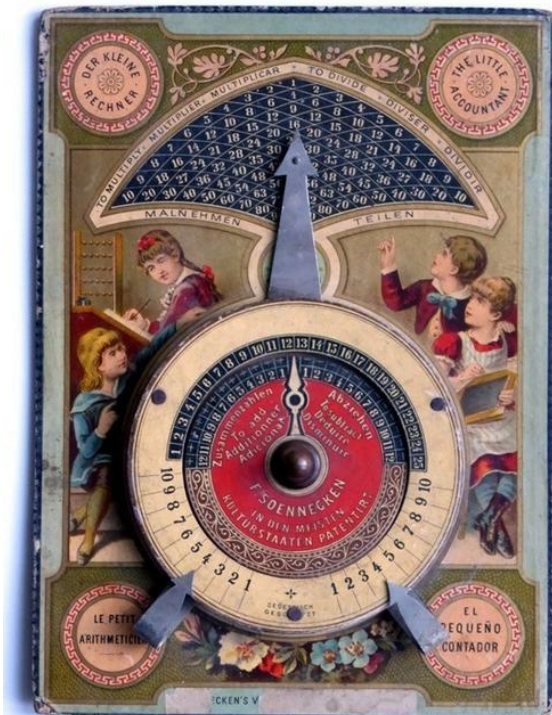

Fig. 4: Soenneckens "small calculator" of 1889 (source: rechnen-ohne-strom.de [5])

In fact, the 'Consul' has German roots [5]. The Friedrich Soennecken publishing house in Bonn had been offering the "small computer" since 1889 (fig. 4), a mechanical "computing machine" with a very similar functional principle, as the patent of Friedrich Soennecken from the year 1889 shows Multiplying and dividing [6], fig. 5.
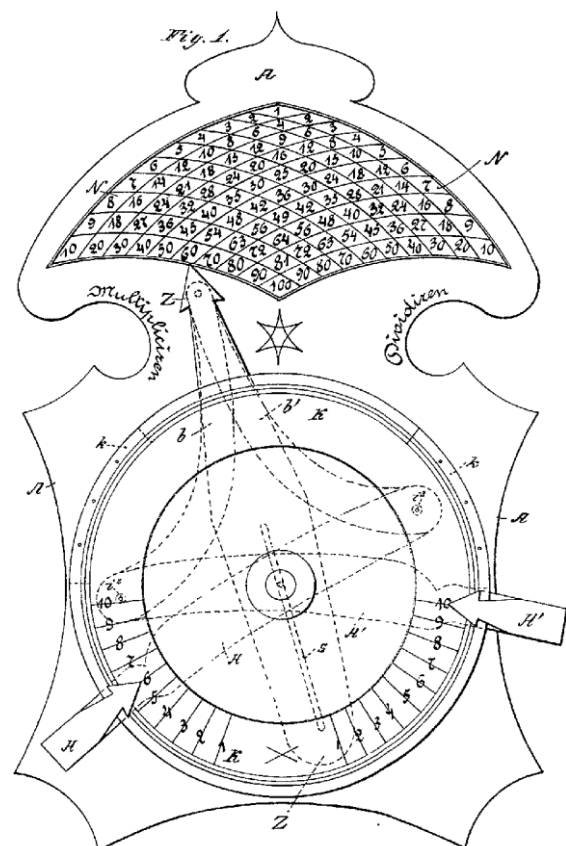

Fig. 5: Patent of Friedrich Soennecken (DRP 51445 from 13/06/1889 [6])

The fischertechnik-Consul

Original replicas of the Consul can be purchased for €13-20. For a true fan of fischertechnik, of course, this is not an option: such a fine mechanism can be realized using fischertechnik – and the money saved can be invested in an expansion of your collection of parts.

First of all we must overcome a few small hurdles – because behind the mechanics lurks, of course, some mathematics.

The Mechanics

The mechanics of the Consul are a little tricky. As can be seen from Friedrich Soenneck's patent drawing and model, the numbers of a two, three, four, etc. series are not on a straight line, as opposed to the Consul. This makes the multiplication or addition table somewhat confusing and reduces the ability of the "small calculator" as a learning aid – after all, it should not replace the mental calculation in your head, but rather help to train it.

But how do we construct the mechanics so that the addition and multiplication tables of the "Consul" form a nicely structured triangle?

The condition is clear: if one holds a "leg" of the arithmetic unit and moves the other to the left or right, the movement of the hands must lie on a straight line. This condition can be expressed mathematically when we draw the geometric structure [7, 8]. Figure 6 shows the geometry of the mechanism with points D, E, F, J, K and L from the patent drawings in figure 2 and 3:
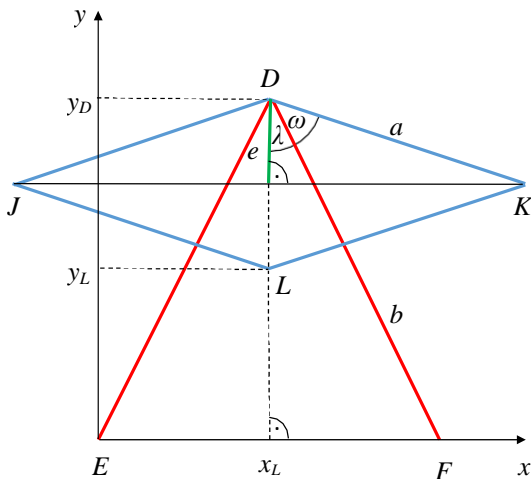

Fig. 6: Geometrical construction of the Educated Monkey

We choose the "arms" of the computing monkey so that upper and lower "arms" are equal (Blue lines, length a) and connect the "legs" (red lines, length b) at a fixed angle

ω with the associated upper arm. We now hold the left "foot" (point E) and move the right "foot" (point F) on the x axis.

We are interested in the motion of the point $L = (x_L, y_L)$:

$$y_L = f(x_L)$$

But how do we get the equation of the function $f(x_L)$?

There again it helps to know a little trigonometry. All the required auxiliary variables – the angle λ and the height e (green line) – are shown in figure 6. We can easily recognize that:

$$y_L = y_D - 2e$$

e and y_D are in turn the catheti of two right-angled triangles, so we have $e = a \cdot cos(\omega + \lambda)$ and $y_D = b \cdot cos\,\lambda$, also it follows:

$$y_L = b \cdot cos\,\lambda - 2a \cdot cos(\omega + \lambda)$$

According to the calculation rules for the cosine of an angle (addition theorem): $cos(w + \lambda) = cos\,\omega \cdot cos\,\lambda - sin\,\omega \cdot sin\,\lambda$, and therefore:

$$y_L = b \cdot cos\,\lambda - 2a \cdot (cos\,\omega \cdot cos\,\lambda \\ - sin\,\omega \cdot sin\,\lambda)$$

Or (re-arranging):

$$y_L = (b - 2a \cdot cos\,\omega) \cdot cos\,\lambda \\ + 2a \cdot sin\,\omega \cdot sin\,\lambda$$

Values a, b and ω are constants, only λ changes when the "foot" F is shifted. We now try to "express" λ by x_L. In the right-angled triangle with the corners E, D and $(x_L, 0)$ this applies: $cos\,\lambda = \sqrt{1 - (sin\,\lambda)^2}$ and $x_L = b \cdot sin\,\lambda$.

This allows us to substitute $sin\,\lambda$ and $cos\,\lambda$ and write y_L as a function of x_L:

$$y_L = (b - 2a \cdot cos\,\omega) \cdot \sqrt{1 - \frac{x_L^2}{b^2}}$$

$$+ 2a \cdot sin\,\omega \cdot \frac{x_L}{b}$$

For $y_L = f(x_L)$ to be a linear function, the root term must be equal to 0 independent of x_L. Now we can formulate our condition mathematically exactly:

The "hands" of the arithmetic unit move exactly on a straight line, if:

$$b - 2a \cdot \cos \omega = 0$$

or

$$b = 2a \cdot \cos \omega$$

We see: In order for the condition to be met, the leg and arm lengths of our arithmetic system must be in a certain relation, which in turn depends on the magnitude of the selected (fixed) angle ω.

Model construction

In order that the legs of our figurines resemble genuine legs, they should have – as with the original Consul – a slightly bent knee joint. For choosing the correct length ratios of legs to (upper) arms, however, we first start from straight legs – the length of a bent lower leg can be easily determined experimentally later.

In the fischertechnik-model of the Consul in Fig. 7, I chose the fixed angle $\omega = 30°$, the (upper) arm length $a = 11$ cm. From this the length of a (straight) leg is calculated as follows:

$$b = 22 \text{ cm} \cdot \cos 30° = 11 \cdot \sqrt{3}$$
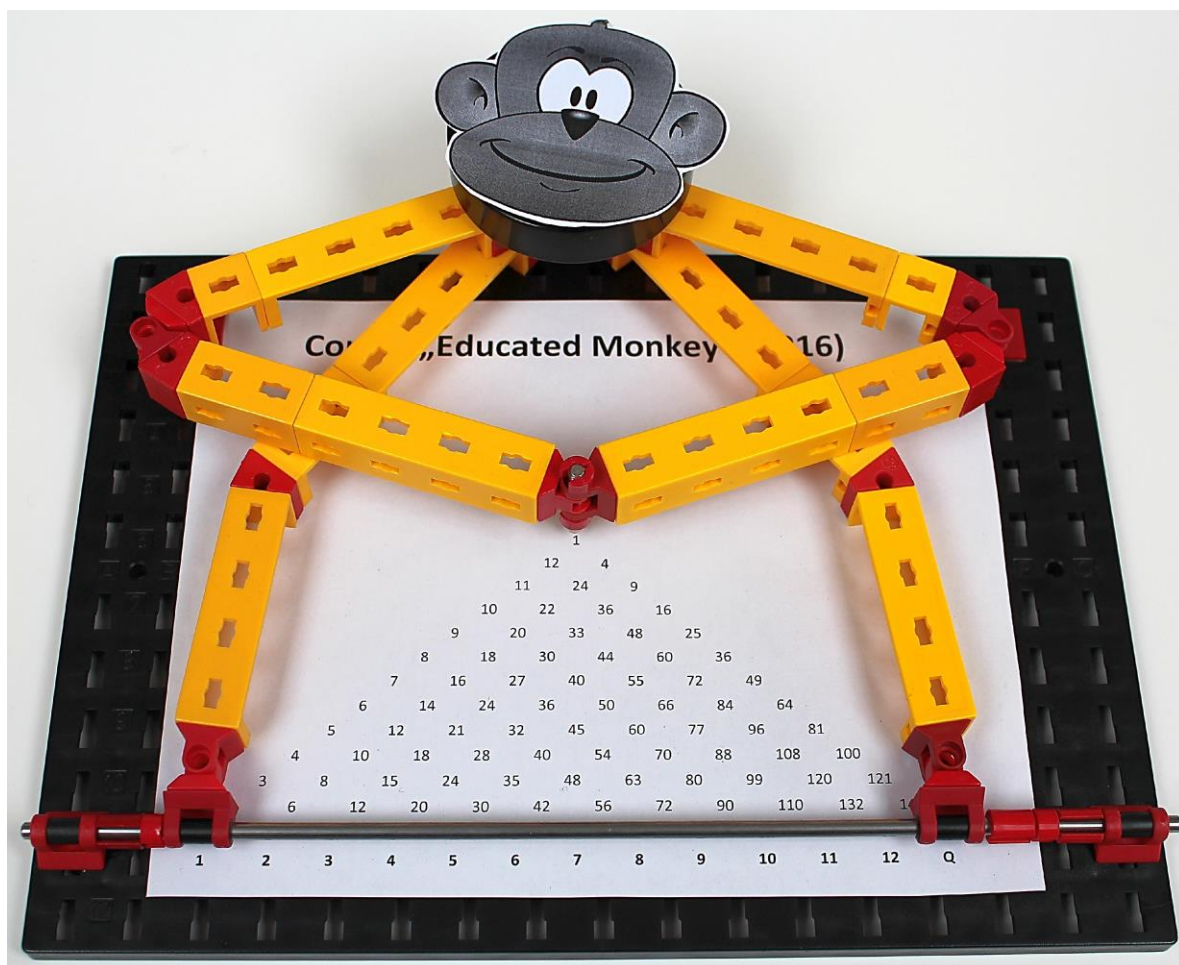$$\cong 19.05 \text{ cm}$$

Fig. 7: Consul, The Educated Monkey made from fischertechnik

Fig. 8: "Head Joint"

The construction of the head joint hidden in Fig. 7 is shown in Fig. 8 above. The arms and legs are joined together in pairs by means of a construction plate 15x30x5 with grooves (38428). You can see the 30° angles, with which the upper arms are angled. Both sides are linked via a 60° angle bracket and a joint claw.
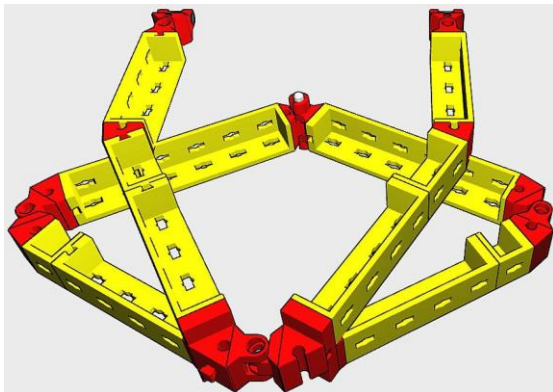

Fig. 9: Overall construction (from "rear")

The 3D construction in Fig. 9 shows the mechanics of the fischertechnik-Consul from behind. The yellow structural part of the upper arms must be shorter than the lower arms because of the head joint construction. Upper and lower arms are connected to each other by means of two angle brackets 30° and a joint claw (as "elbow").

In Fig. 10, you see the two hands of the arithmetic unit holding a metal axle 30 as a "pointer", and the feet which can be moved on the metal cube claws (31436) with bearing sleeve (36819) on a metal axis 260 (107436).
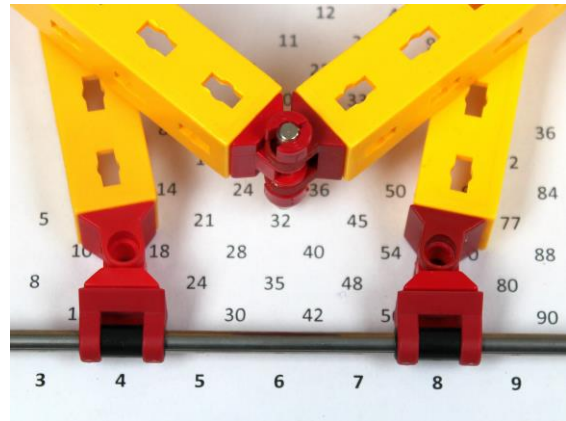

Fig. 10: "Feet" and "Hands"

If a 30° angle is used as a knee joint in the leg after 10.5 cm (seven building heights), the leg of the leg must be shortened by 0.75 cm (= ½ height) (Fig. 9).

A metal axle 60 with clip 10, on which the head of the monkey is mounted, is inserted through the head joint. A wheel 23 (36581), a roller wheel (35386) or a rotary disc 60 (31019) can alternatively be used for this purpose.

Fig. 11: "Head" (underside)

If you want, you can stick the likeness of a monkey on the top − for example, the one shown in Fig. 12, is available for a [Free Download](#) under a Public Domain License.
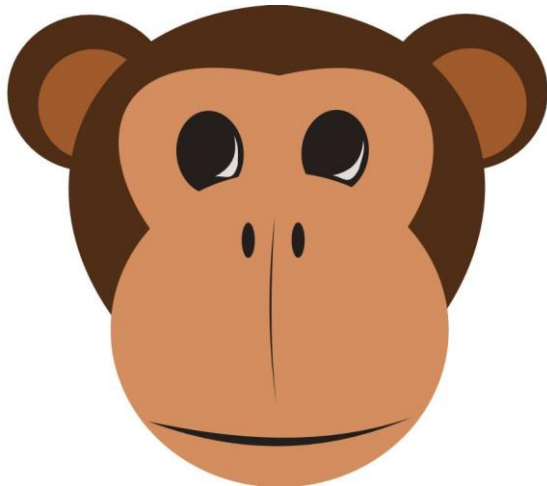
ft:pedia

Fig. 12: Monkey Head (Public Domain)

The multiplication scheme

Now the multiplication scheme is still required in the background. It is very easy to create a text program by writing a "1" centered in the middle of the sheet and then entering the numbers one by one, each separated by a tab. In the diagonals from the upper left to the lower right, the 2s, 3s, 4s up to the 12th row – and in the far right all squared numbers from 1-144 (see figures 2, 3).

Then you select the total number of triangles and choose the tab length so that the bottom line has a width of 16 cm. Finally, it adjusts the line spacing so that the number pyramid is exactly 7 cm high. A line with the factors of 1-12 followed by "□" for the number squared ("Quadrieren" in German, therefore the letter "Q" is used) is inserted at a distance (approx. 1 cm).

In this way, an addition scheme can also be created – since the addition operation is also commutative; our Consul works just as well as an addition tool [8].

Downloads

I have provided the Multiplication and Addition Tables for download.

If you are using the fischertechnik designer, you will find a 3D-Building Instructions in the download area of the ft-community.

Of course, a fischertechnik-Consul with other proportions can also be constructed – simply experiment with the angle ω and the upper arm length a.

I wish you much pleasure with your fischertechnik-Consul calculator!

References

[1] Flash-Simulation (http://www.tan-gram.de/consul.pl)

[2] Reinhard Atzbach: *Consul the Educated Monkey for Windows*, 2001.

[3] William H. Robertson: *Toy*. US-Patent No. 1188490, 27/6/1916.

[4] William H. Robertson: *Calculation Device*. US-Patent No. 1286112, 26/11/1918.

[5] Wilfried Denz: *Rechenaffen*. Rechnen-ohne-Strom.de.

[6] Friedrich Soennecken: *Rechenvor-richtung zum Vervielfachen und Theilen*. German Patent No. 51445, 13/6/1889.

[7] Dörte Haftendorn: *Educated Monkey oder "Der rechnende Affe"*. 19/9/2008.

[8] Sidney J. Kolpas, Gary R. Massion: *Consul, the Educated Monkey*. In: The Mathematics Teacher, Vol. 93, No. 4, April 2000, S. 276-279.

Translator's Note: Many of the web links, from the original article, are now invalid so they have therefore been updated. The next page provides my added comments and illustration.

My need to translate this article stemmed from the desire to understand the mathematics involved and an additional desire to provide an example of how mathematics is used in engineering design. Of course the same result could have been achieved by "Trial and Error". As mentioned in the article, one can still play around with different values of ω, a and b, perhaps removing the pointer and replacing it with the insert of a pen, to trace the movement of the pointer across the paper as the right or left leg is moved accordingly.

If you have a child who is learning their times table, then constructing this project could be of great benefit to them. To help those learning their times table it is best to position the squares of numbers as shown below. Were it possible to position both feet on the same number, then the bottom of the triangle would in fact be where the pointer would position itself. To illustrate this further, please download and run the Windows version of *Consul the Educated Monkey* from Reinhard Atzbach [2].

Enjoy… Peter King - www.procontechnology.com.au